Self-consistent dynamical models with central black holes

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- Dynamical evidence of intermediate-mass black holes (IMBH) in dense stellar systems crucially relies on the interpretation of their velocity dispersion profile [1], especially in the central regions.
- Three key ingredients:
  1. Lowered isothermal phase space density function [2]

$$f(E) = \begin{cases} Ae^{-aE_0}\left[e^{-a(E-E_0)} - 1\right], & \text{for } E \leq E_0 \\ 0, & \text{for } E > E_0. \end{cases}$$

  2. Hydrostatic equilibrium [3]

$$\frac{\mathrm{d}\psi}{\mathrm{d}\hat{r}} = -\frac{9\mu}{4\pi\hat{r}^2} - \frac{9}{4\pi\hat{r}^2}\int_\delta^{\hat{r}} 4\pi s^2 \frac{\hat{\rho}(\psi)}{\rho_0}\mathrm{d}s.$$

  3. Self-consistency via the Poisson equation

$$\begin{cases} \frac{\mathrm{d}^2\psi}{\mathrm{d}\hat{r}^2} + \frac{2}{\hat{r}}\frac{\mathrm{d}\psi}{\mathrm{d}\hat{r}} = -9\frac{e^{\psi}\gamma\left(\frac{5}{2},\psi\right)}{e^{\Psi}\gamma\left(\frac{5}{2},\Psi\right)} = -9\frac{\hat{\rho}(\psi)}{\hat{\rho}(\Psi)}, \\ \psi(\delta) = \Psi, \\ \psi'(\delta) = -\frac{9\mu}{4\pi\delta^2}. \end{cases}$$

- Three parameters: Minimum radius δ, IMBH mass μ, and central concentration Ψ.

An interesting transition

Along a sequence of models with fixed Ψ and δ we note

- A rapid expansion and contraction of the spatial extent of the system.
- The stellar mass quickly becomes negligible with respect to the black hole mass.

Two immediate questions emerge:

1. Where does this transition occur?
2. Is there any physical meaning to solutions on either side of this transition?

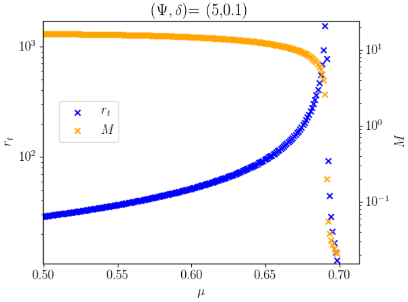

Figure 1: A plot of the spatial extent of the system rt , and the total stellar mass of the system in dimensionless units, for a sequence of models with increasing black hole masses.

Asymptotic explanation

- Assume $\Psi - \frac{9\mu}{4\pi\delta} = \alpha\delta^2$, and let $\hat{r} = \delta r_1$.

$$\nabla^2_{r_1}\psi^{(\mathrm{I})} = -9\delta^2 \frac{\hat{\rho}(\psi^{(\mathrm{I})})}{\hat{\rho}(\Psi)},$$
$$\psi^{(\mathrm{I})}(1) = \Psi,$$
$$\psi^{(\mathrm{I})\prime}(1) = -\Psi + \alpha\delta^2.$$

- For sufficiently large r_1 we can write down $\psi^{(\mathrm{I})}$ to second order

$$\psi^{(\mathrm{I})} = \frac{\Psi}{r_1} + \delta^2\left(a_0 + \frac{a_1}{r_1} + \frac{72}{5\hat{\rho}(\Psi)}\Psi^{\frac{5}{2}} r_1^{-\frac{1}{2}}\right).$$

where $a_0 = \alpha - \frac{9}{\hat{\rho}(\Psi)}\int_1^\infty \frac{1}{t^2}\int_1^t s^2 \hat{\rho}\left(\frac{\Psi}{s}\right) \mathrm{d}s\, \mathrm{d}t.$

- Expansion breaks down for $r_1 = O(\delta^{-2})$ requiring further asymptotic regions, or $\psi^{(\mathrm{I})}$ reaches zero first. This suggest a change in behaviour when $a_0 = 0$, supported by Fig. 2.

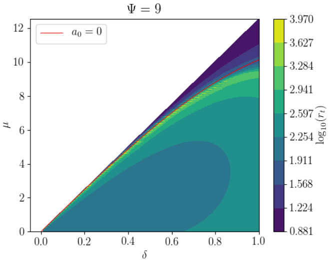

Figure 2: Contour plot of the tidal radius across a 2-D slice of the parameter space. Shown in red is the proposed condition for the transition identified in Fig. 1.

- **The models are characterised by substantial central slopes**, both in the density and the velocity dispersion profiles. The properties of the central regions may captured by asymptotic solutions.
- **The equilibria change structure as the black hole mass is increased past the critical point** indicated in Fig.2. This results in low mass cluster with no visible core in the profile, unlike the lower IMBH mass counterparts.

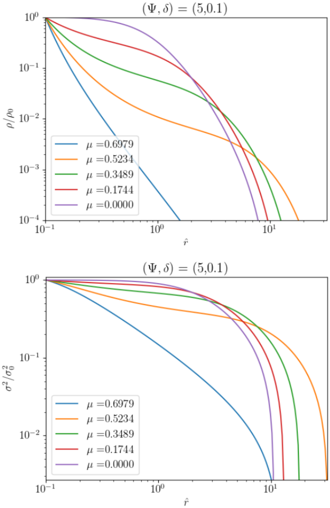

Figure 3: Intrinsic profiles for the density (top) and velocity dispersion (bottom).

References

1. Noyola et al, 2010, ApJ, 719, L60
2. King 1966, AJ, 71, 64
3. Huntley & Saslaw, 1975, ApJ, 199, 328
4. Miocchi, MNRAS, 2007, 381, 103